resolve

Mathematical Concepts

Jakob Roth, MPA Garching

\+ Andreas Popp, ESO Garching

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Bayesian Imaging

Bayes' theorem

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\mathcal{P}(s)}{\mathcal{P}(d)}$$

- Data: d (e.g. visibilities)
- Signal: s (e.g. sky brightness)

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- How to compute the posterior $\mathcal{P}(s|d)$?

Bayesian Imaging – Software

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NIFTy

- `https://gitlab.mpcdf.mpg.de/ift/nifty`
- Prior Models
- Inference Algorithms

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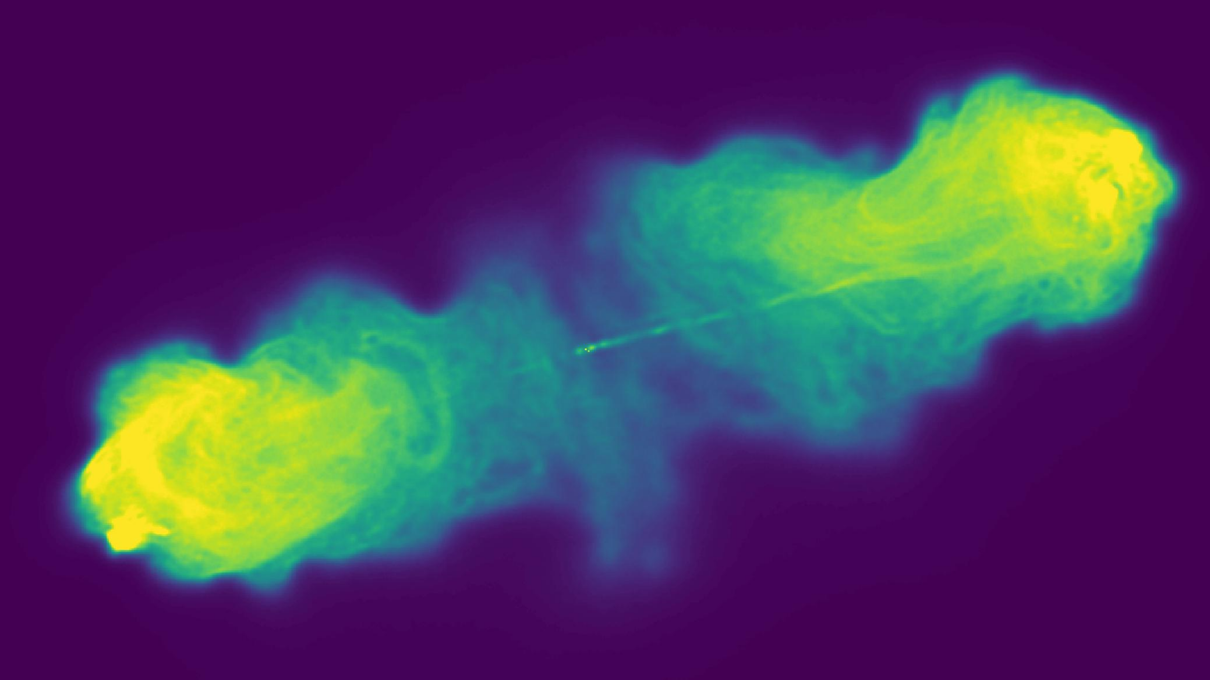
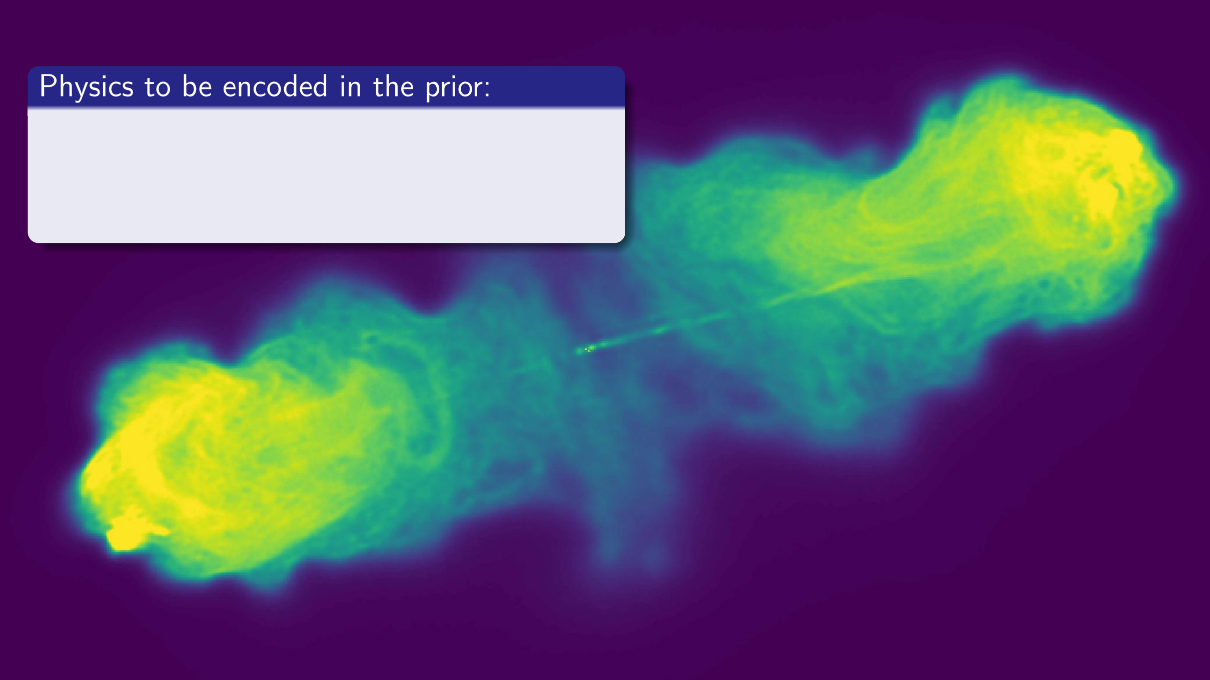
`resolve`

- `https://gitlab.mpcdf.mpg.de/ift/resolve`
- Handling radio interferometric data
- Measurement equation

How to choose the prior $\mathcal{P}(s)$?



Physics to be encoded in the prior:



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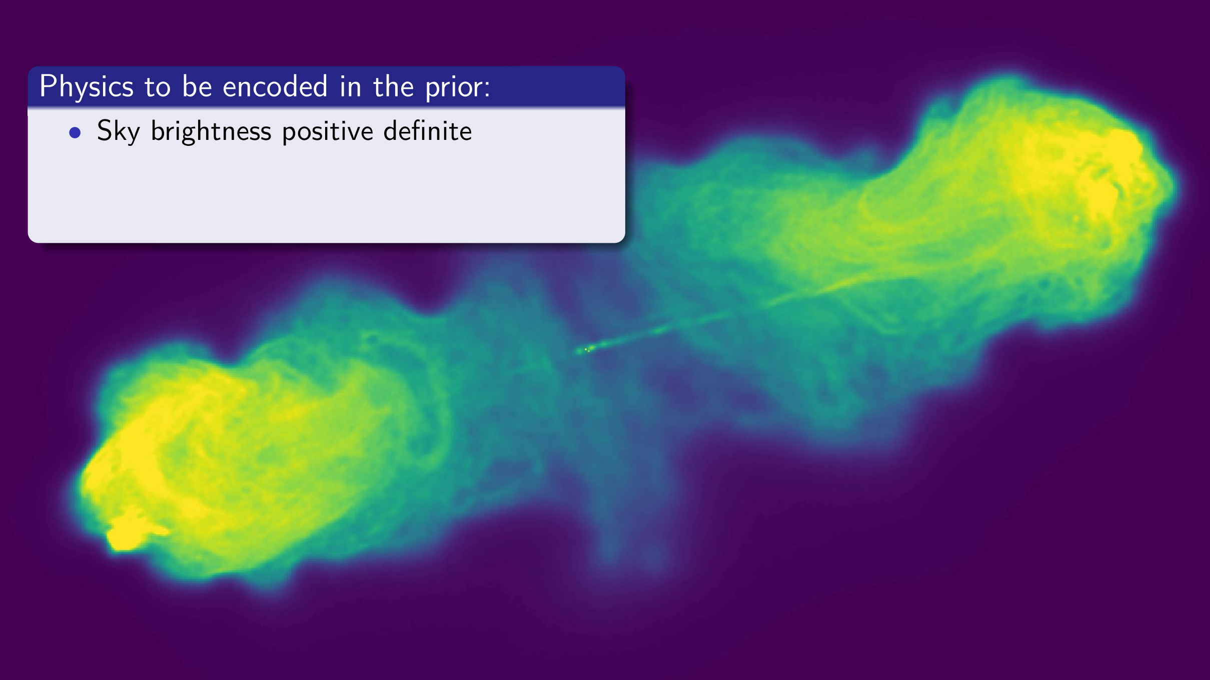

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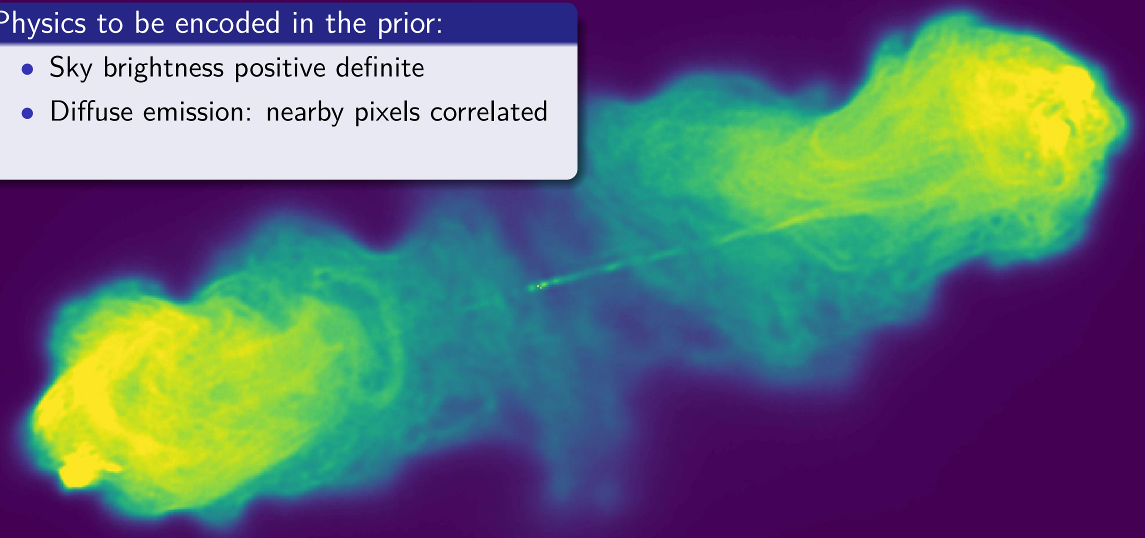

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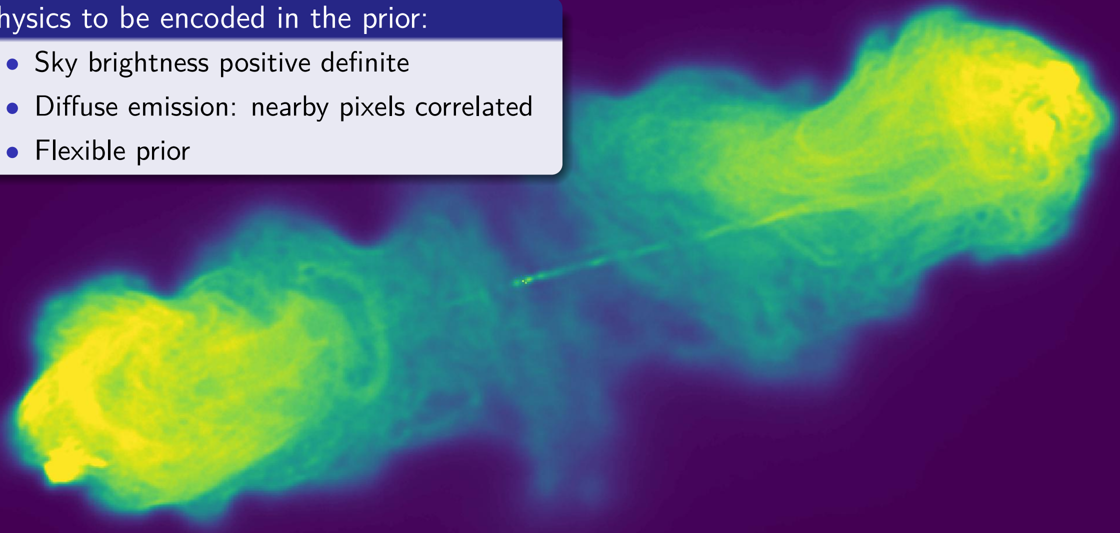

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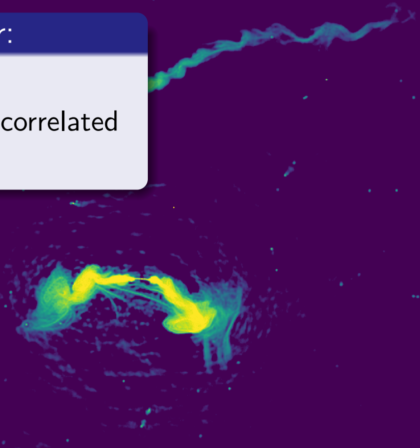

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Complex prior distribution

$\rightarrow$ Prior as generative model

Prior – Generative Models

Standardized Generative Model

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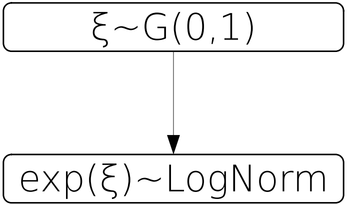

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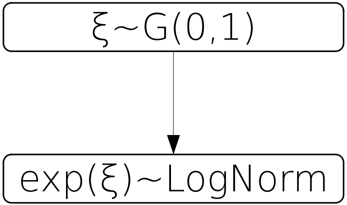

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$\surd$ Positivity

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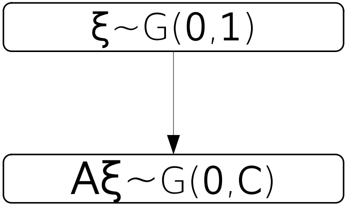

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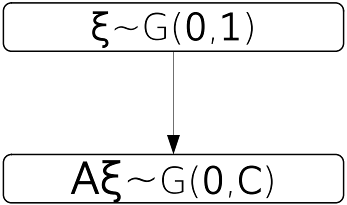

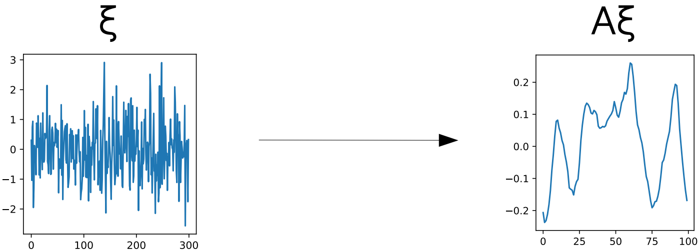

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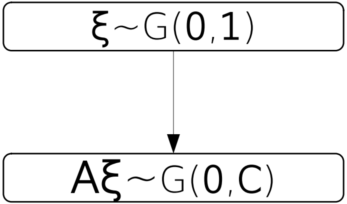

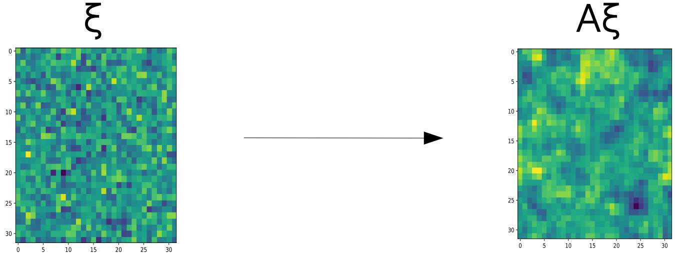

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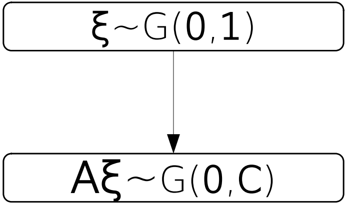

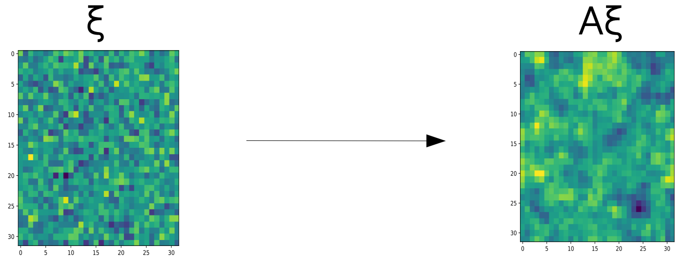

√ Correlation

Prior – Generative Models

Example: Correlated Log Normal

- $\boldsymbol{\xi} \sim \mathcal{G}(\mathbf{0}, \mathbb{I})$
- $s(\xi) = \exp(A\boldsymbol{\xi})$ with $A = \sqrt{C}$
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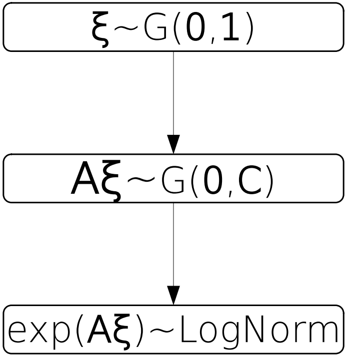

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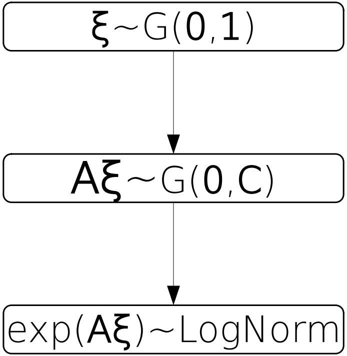

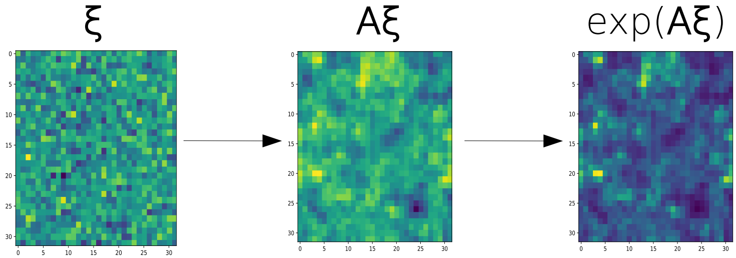

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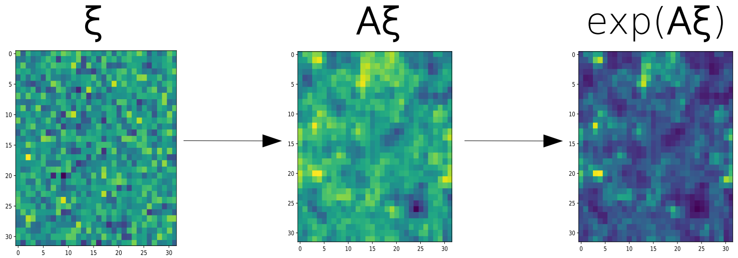

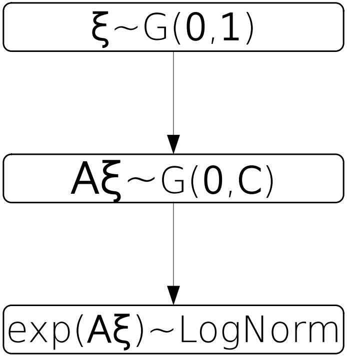

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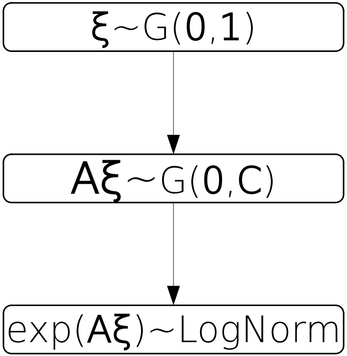

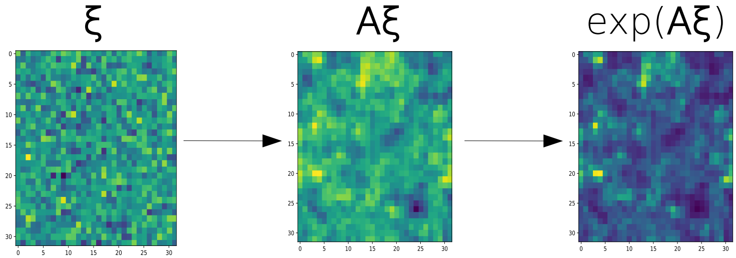

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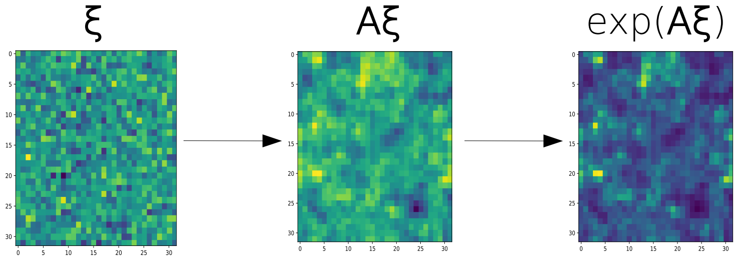
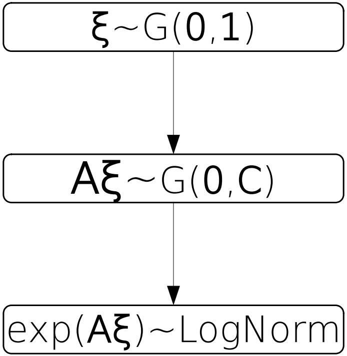
Prior – Generative Models

statistical homogeneity and isotropy





Prior – Generative Models

statistical homogeneity and isotropy

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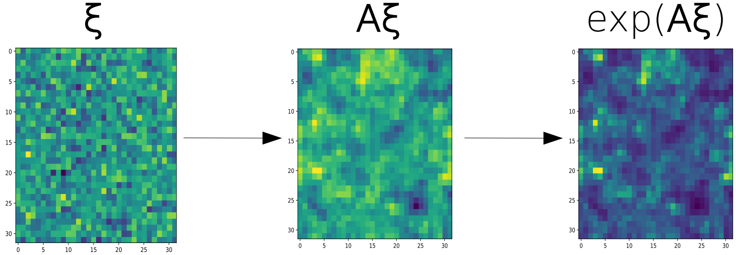

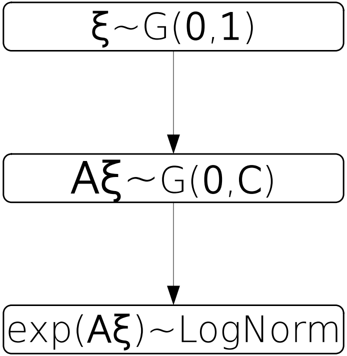

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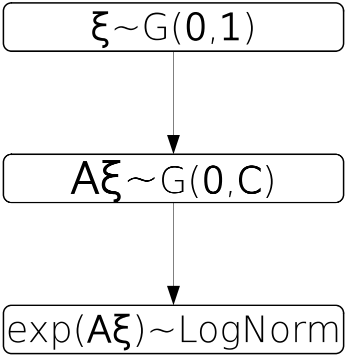

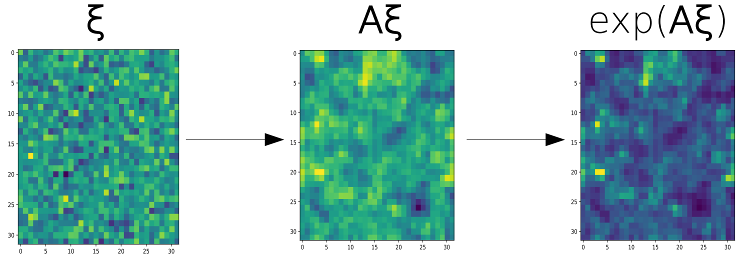

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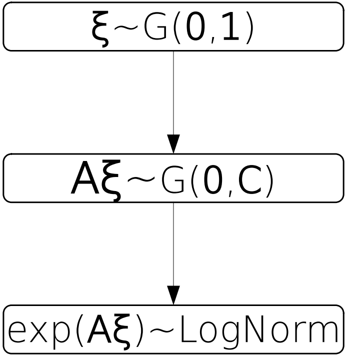

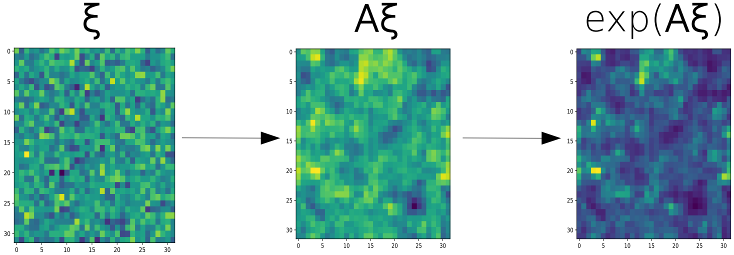

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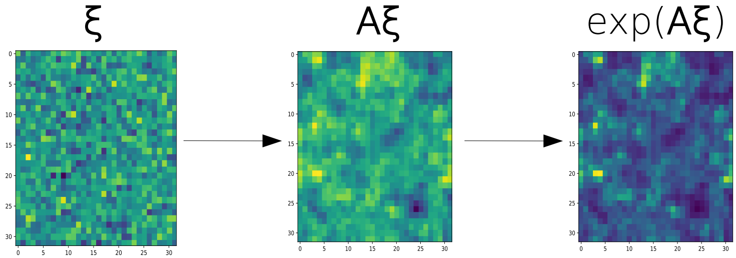
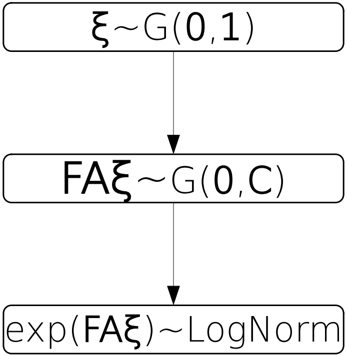




Prior – Generative Models

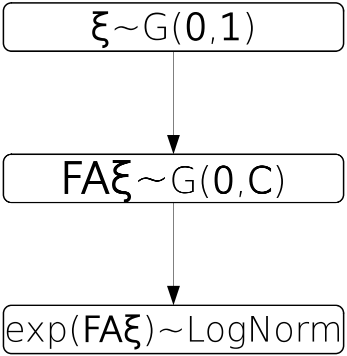
```python
import numpy as np
import nifty8 as ift

def A_spec(k):
  return 1./(20. + k**2)

...

HT = ift.HarmonicTransformOperator(...)
sky = ift.exp(HT(ift.DiagonalOperator(A)))

xi = ift.from_random(sky.domain)
sample = sky(xi)
```



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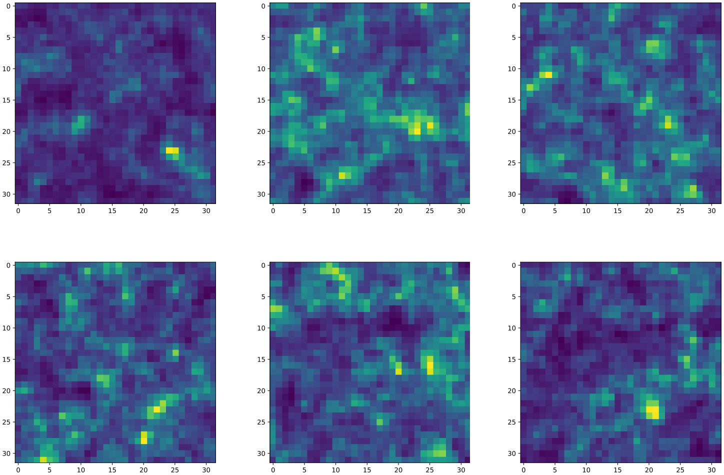


$$\xi \sim \mathcal{G}(0,1)$$

↓

$$FA\xi \sim \mathcal{G}(0,C)$$

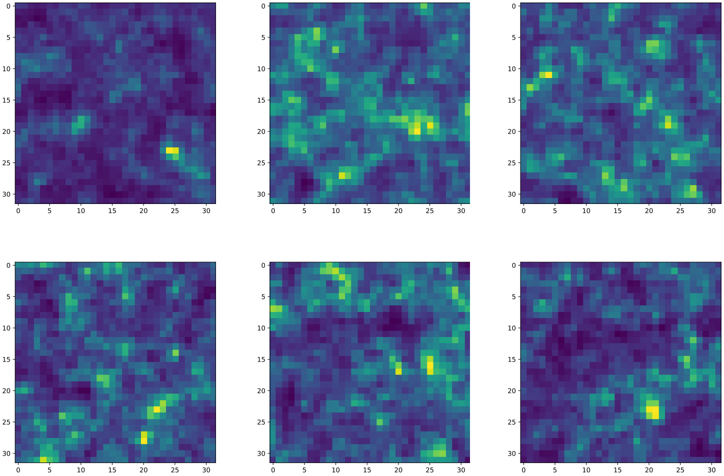
↓

$$\exp(FA\xi) \sim \text{LogNorm}$$

√ Positivity

√ Correlation

Prior – Generative Models



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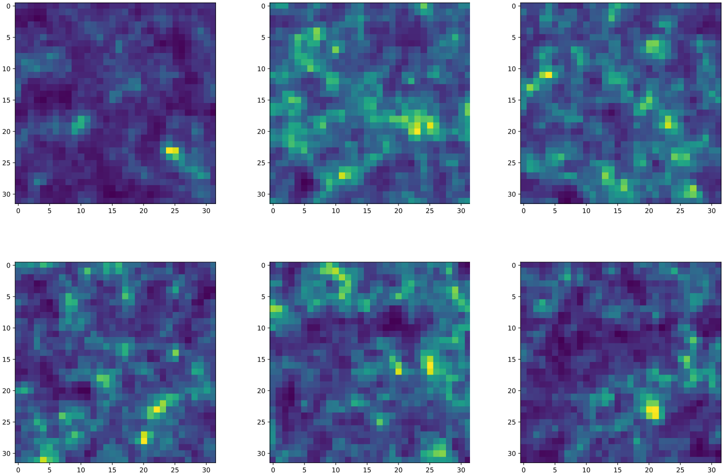

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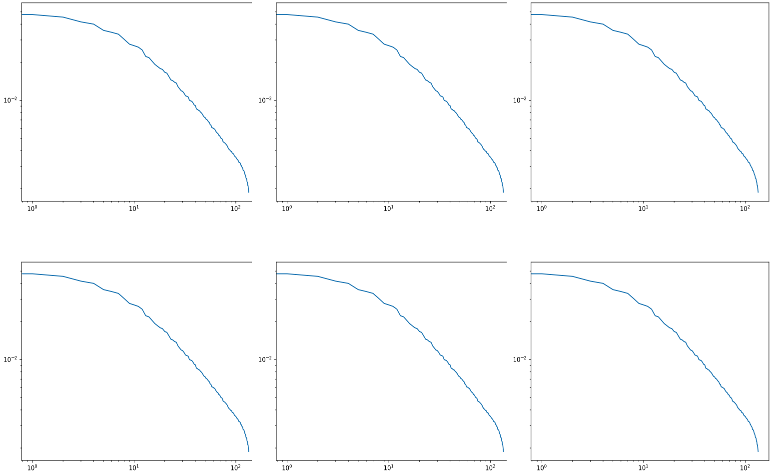

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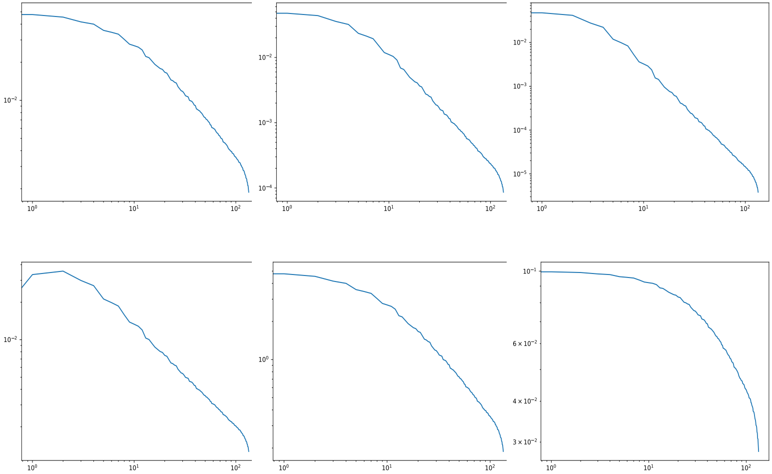

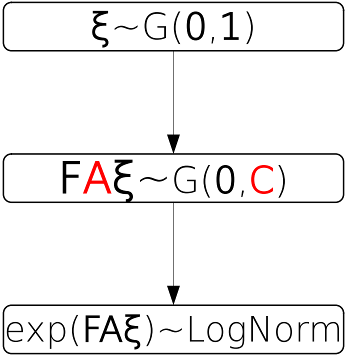

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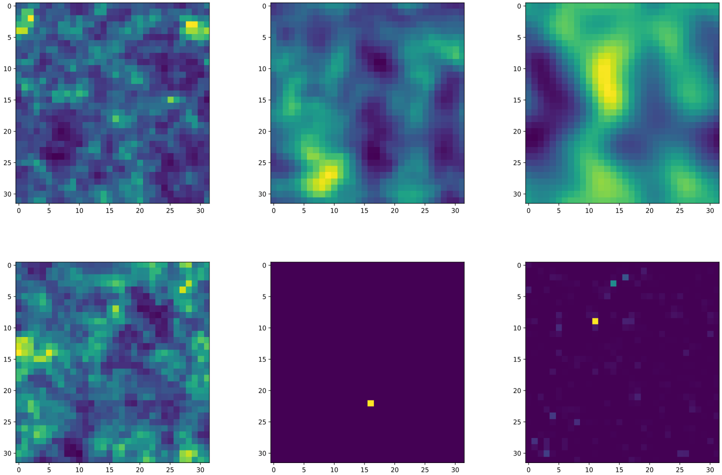

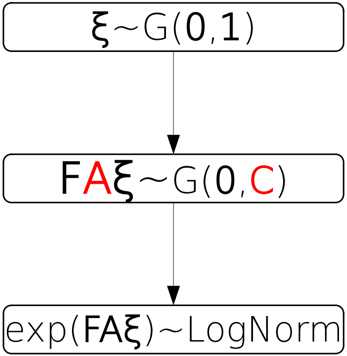

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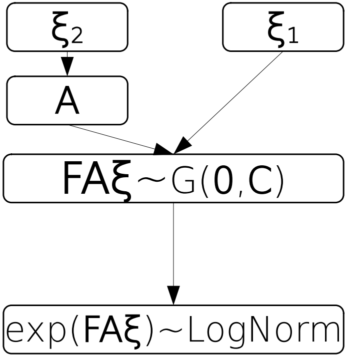

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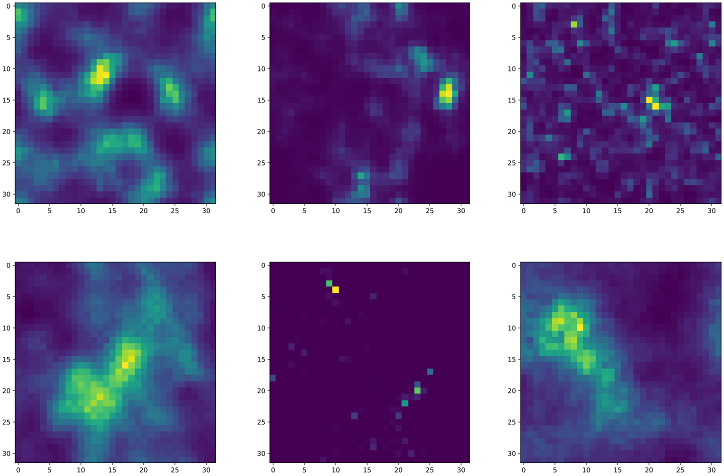

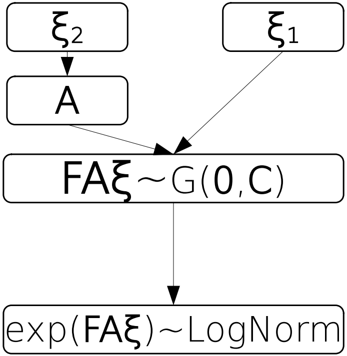

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- √ Correlation
- √ Flexible

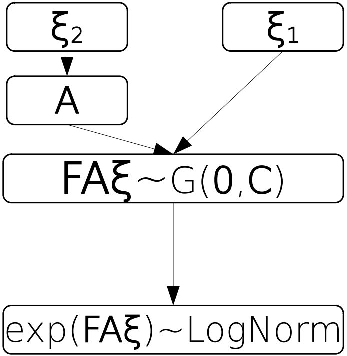
Prior – Generative Models

```
import numpy as np
import nifty8 as ift

x_space = ift.RGSpace([100, 100])
cfm = ift.SimpleCorrelatedField(x_space, ...)

sky = ift.exp(cfm)

xi = ift.from_random(sky.domain)
sample = sky(xi)
```



Prior – Generative Models

```python
import numpy as np
import nifty8 as ift

x_space = ift.RGSpace([100, 100])

args = {
'offset_mean': 0.,
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'fluctuations': (1.5, 0.5),

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'flexibility': None,
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cfm = ift.SimpleCorrelatedField(x_space, **args)
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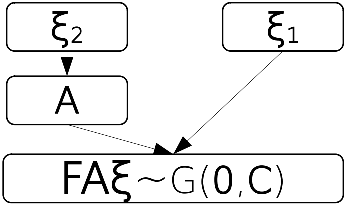

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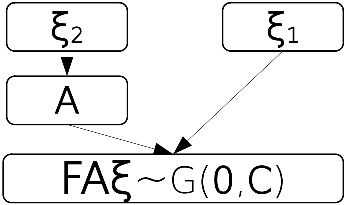

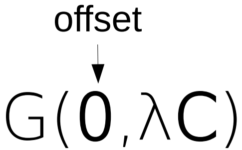

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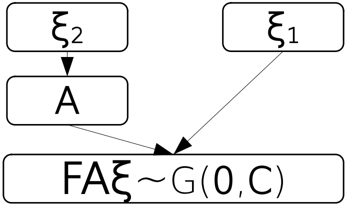

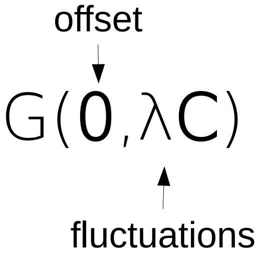

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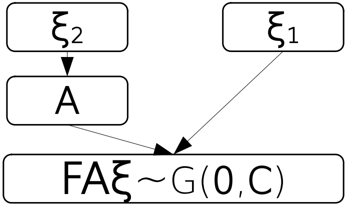

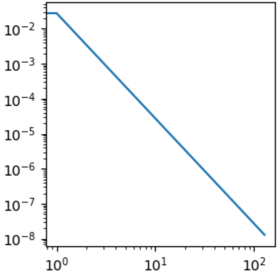

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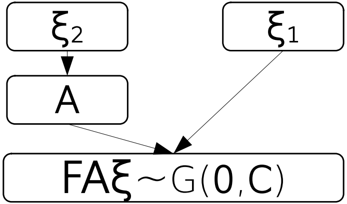

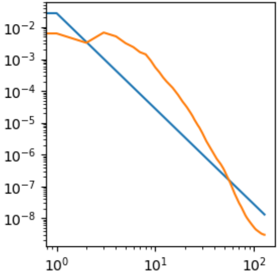

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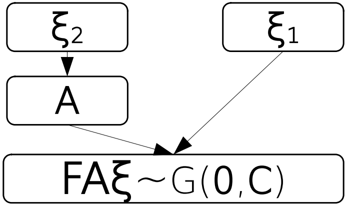

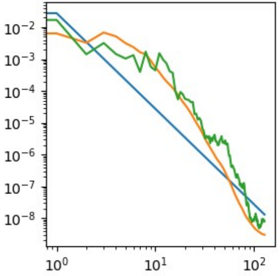

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\mathcal{P}(s)}{\mathcal{P}(d)}$$

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$$\Downarrow$$

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$$\mathcal{P}(\xi|d) = \frac{\color{red}{\mathcal{P}(d|s(\xi))}\color{teal}{\mathcal{P}(\xi)}}{\mathcal{P}(d)}$$

How to construct the likelihood $\mathcal{P}(d|s)$?

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Response

$$\widetilde{V} = \int \frac{dldm}{n(l,m)} G_p(l,m,t) I(l,m) G_q^*(l,m,t) e^{-ik\cdot x}$$

- Sky brightness: I
- Antenna gain: G
- Model visibilities: $\widetilde{V}$

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Measurement equation:

$$V = RI + n$$

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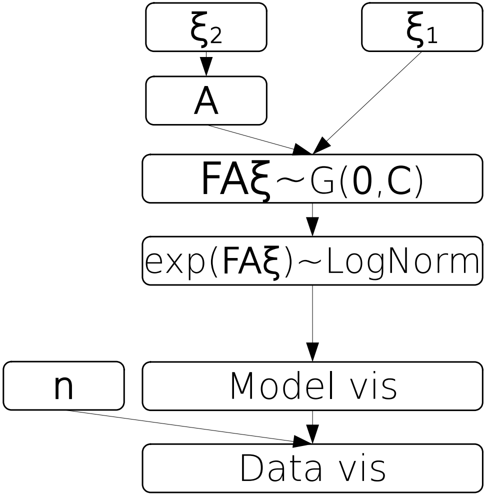


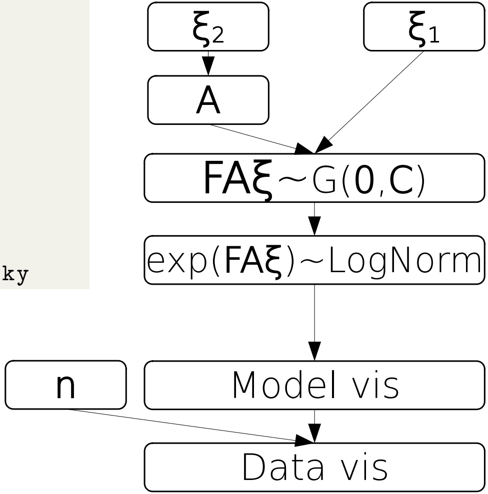
How to construct the likelihood $\mathcal{P}(d|s)$?

```python
import numpy as np
import nifty8 as ift
import resolve as rve

cfm = ift.SimpleCorrelatedField(...)
sky = ift.exp(cfm)

R = rve.InterferometryResponse(...)

log_lh = ift.GaussianEnergy(...) @ R @ sky
```



Inference

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cfm = ift.SimpleCorrelatedField(...)
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R = rve.InterferometryResponse(...)
log_lh = ift.GaussianEnergy(...) @ R @ sky

samples = ift.optimize_kl(log_lh, ...)
```

MGVI – Metric Gaussian Variational Inference[1]

[1] Jakob Knollmüller and Torsten A. Enßlin. "Metric Gaussian Variational Inference". In: (Jan. 30, 2019). arXiv: 1901.11033v3 [stat.ML].

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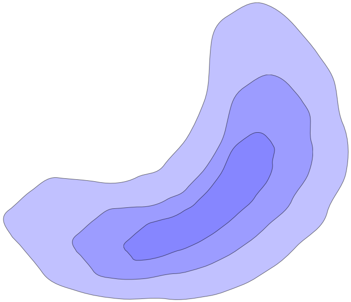

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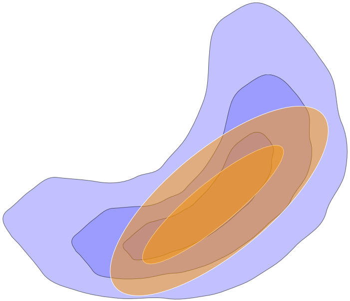

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  Approximate $P(\xi|d)$ with $Q(\xi) = \mathcal{G}(\bar{\xi}, \Xi)$

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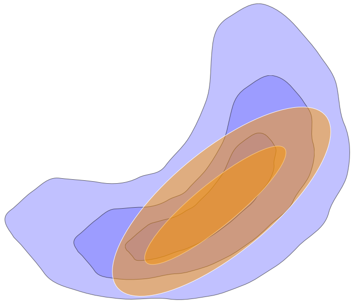

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How to determine $\bar{\xi}$ and Ξ:

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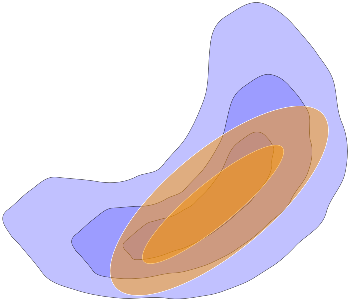

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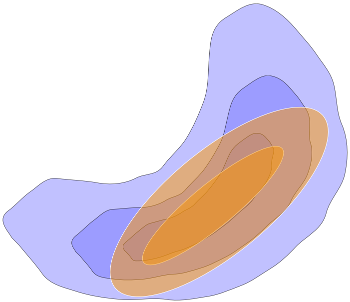

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- $\bar{\xi} = \mathrm{argmin}_{\bar{\xi}}(\mathcal{D}_{KL}(Q||P))$
- $\Xi^{-1}(\bar{\xi}) = \left\langle \frac{\partial^2 \mathcal{H}(d,\bar{\xi})}{\partial\bar{\xi}\partial\bar{\xi}^\dagger} \right\rangle_{P(d|\bar{\xi})}$
  with $\mathcal{H} = -\ln(P)$
- Linear scaling with dimensions of ξ

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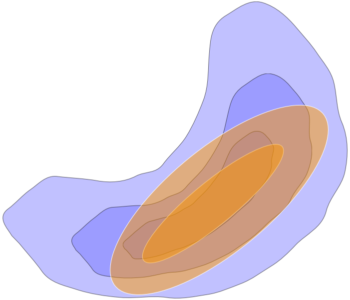
GeoVI – Geometric Variational Inference[2]



[2] Philipp Frank, Reimar Leike, and Torsten A. Enßlin. "Geometric Variational Inference". In: *Entropy* 23.7 (2021). ISSN: 1099-4300. DOI: 10.3390/e23070853. URL: https://www.mdpi.com/1099-4300/23/7/853.

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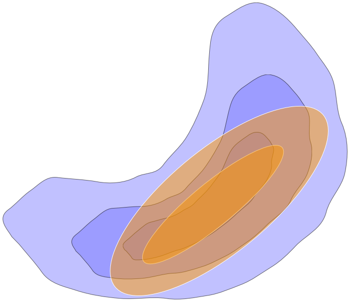

- Coordinate transformation in latent space

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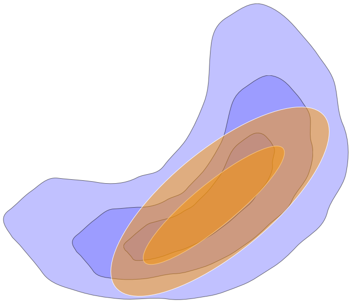

- Coordinate transformation in latent space
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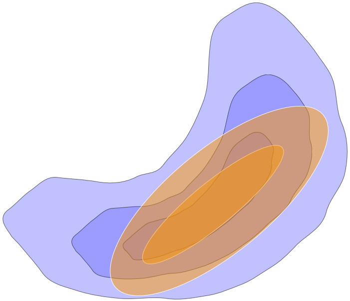

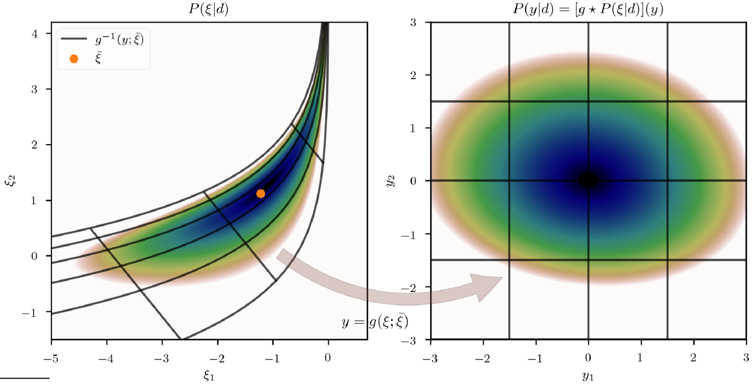

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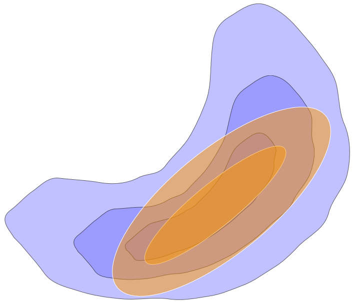

- Coordinate transformation in latent space
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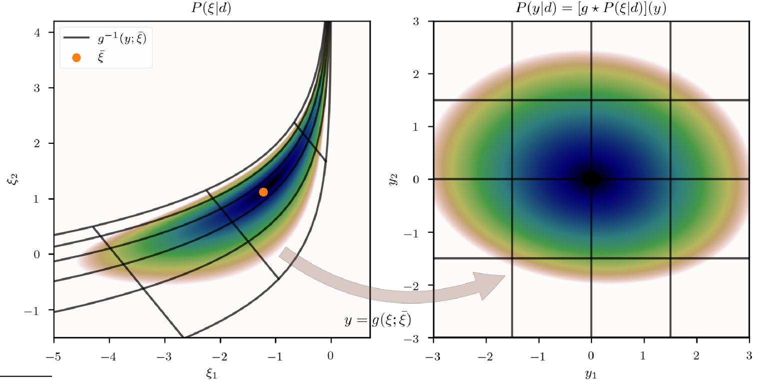
- Also linear scaling with number of dimensions



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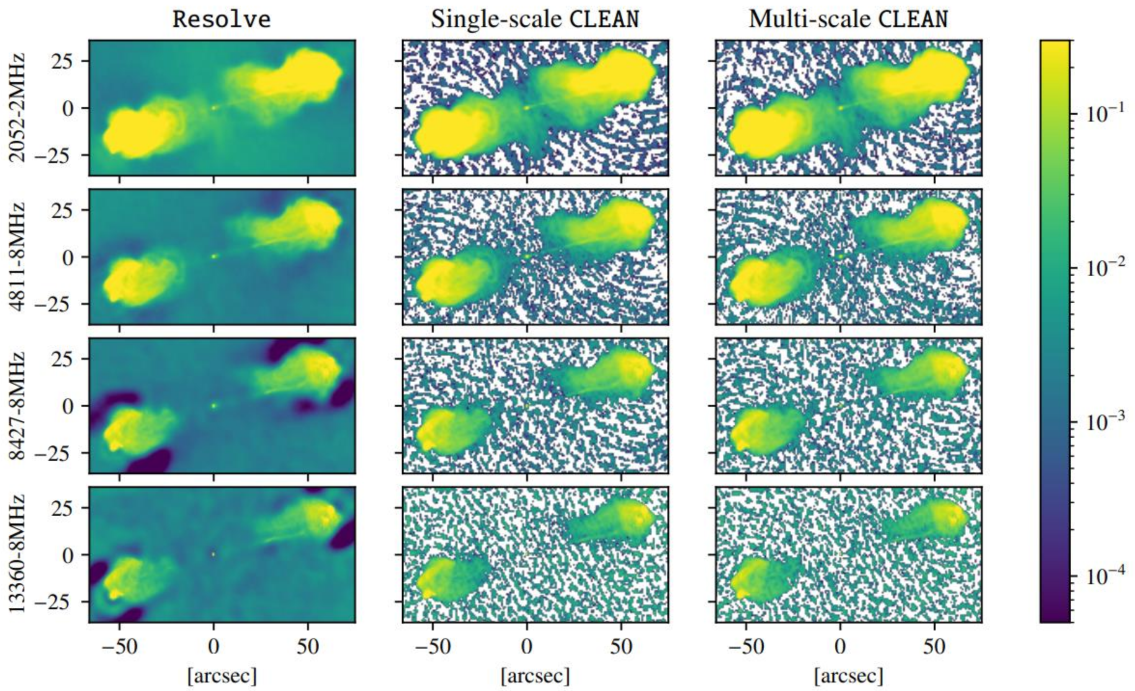
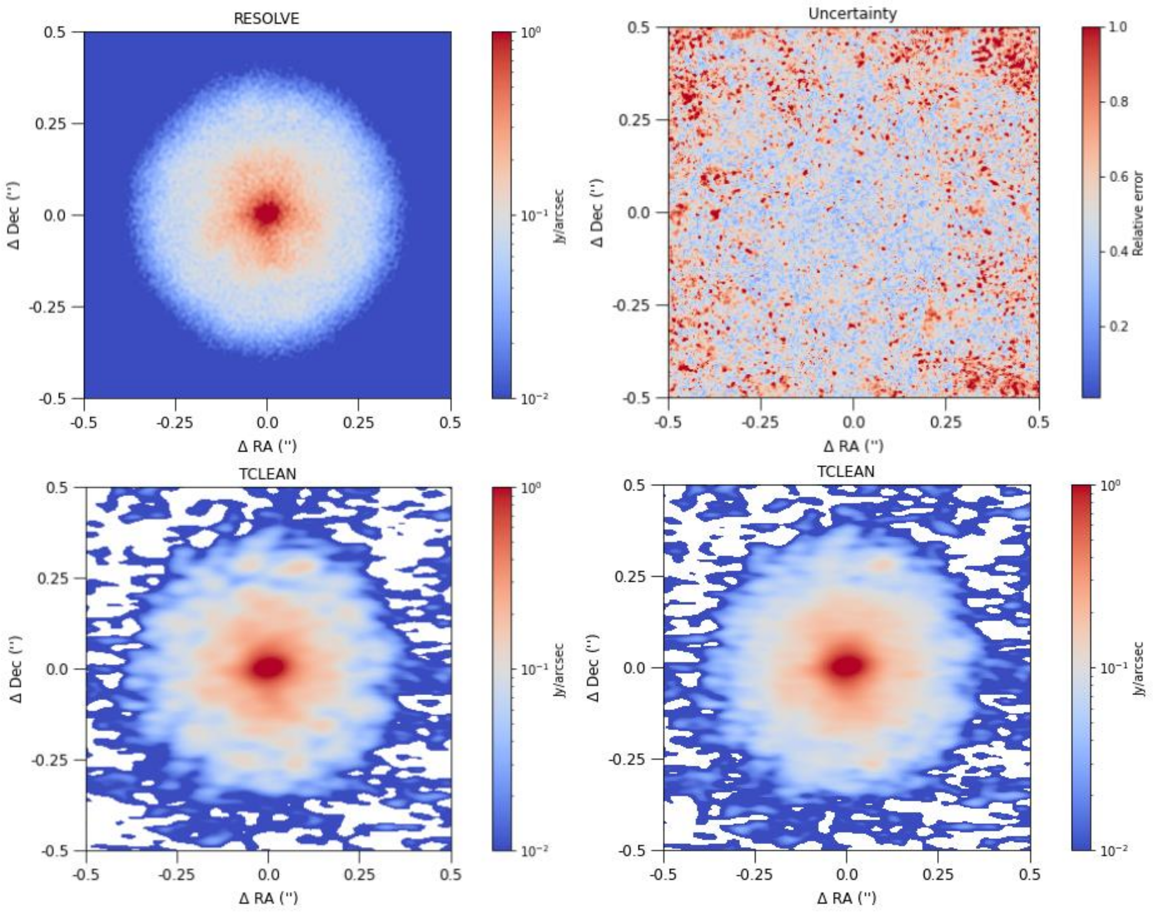
Imaging (VLA)

Arras et al., 2021



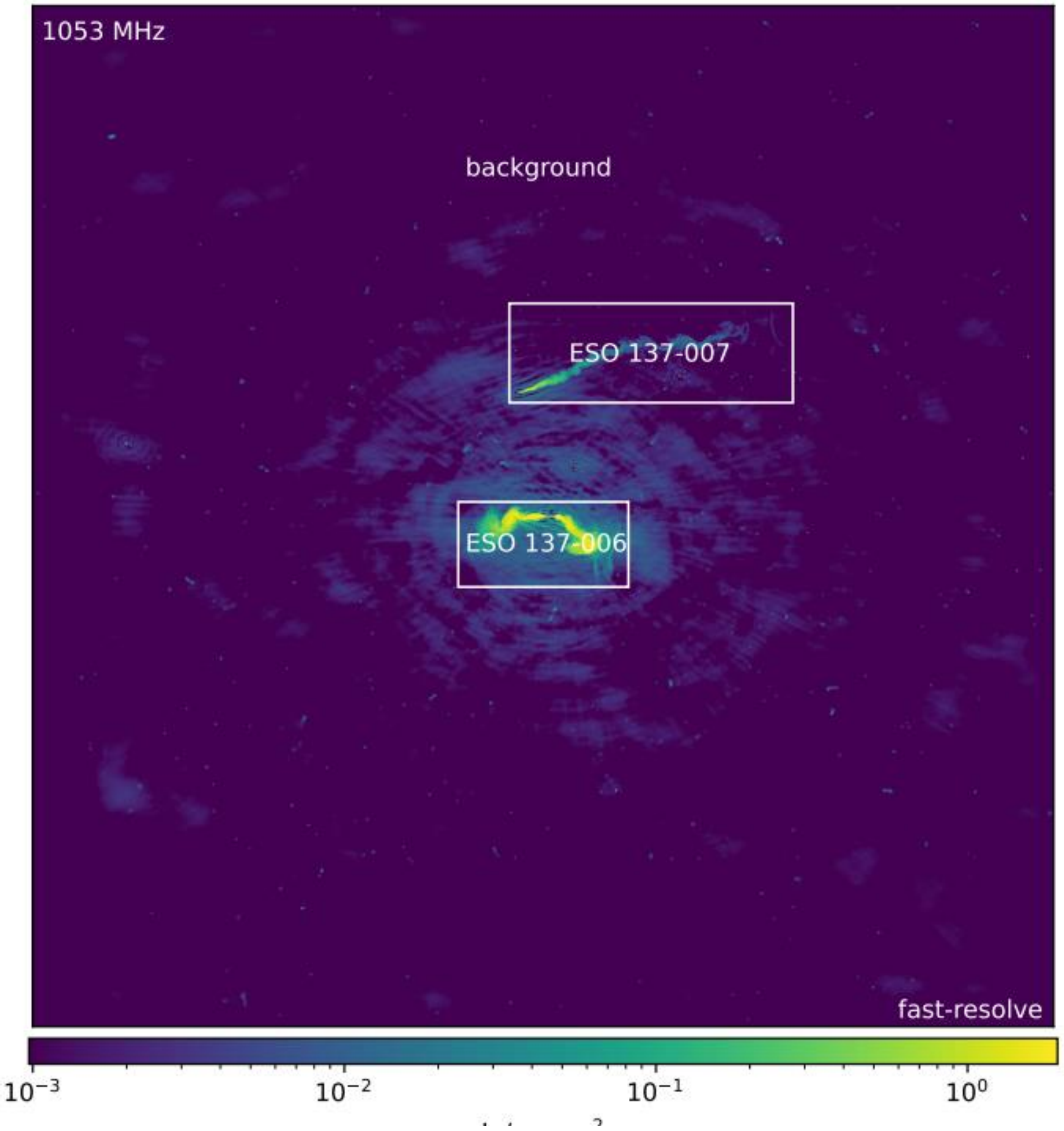
Imaging (ALMA)

Tychoniec et al., 2022



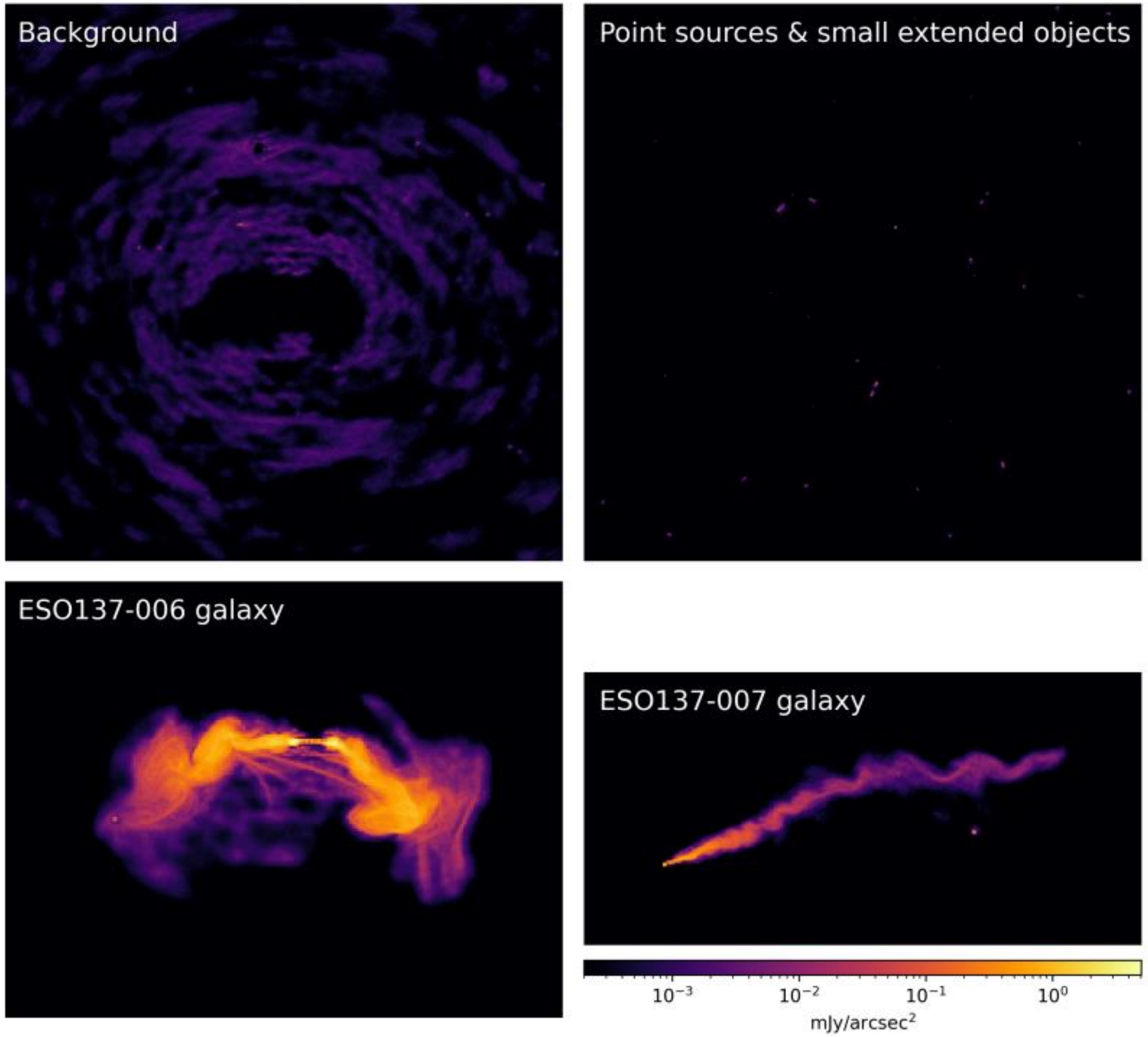
Imaging (MeerKAT)

Roth et al., 2024



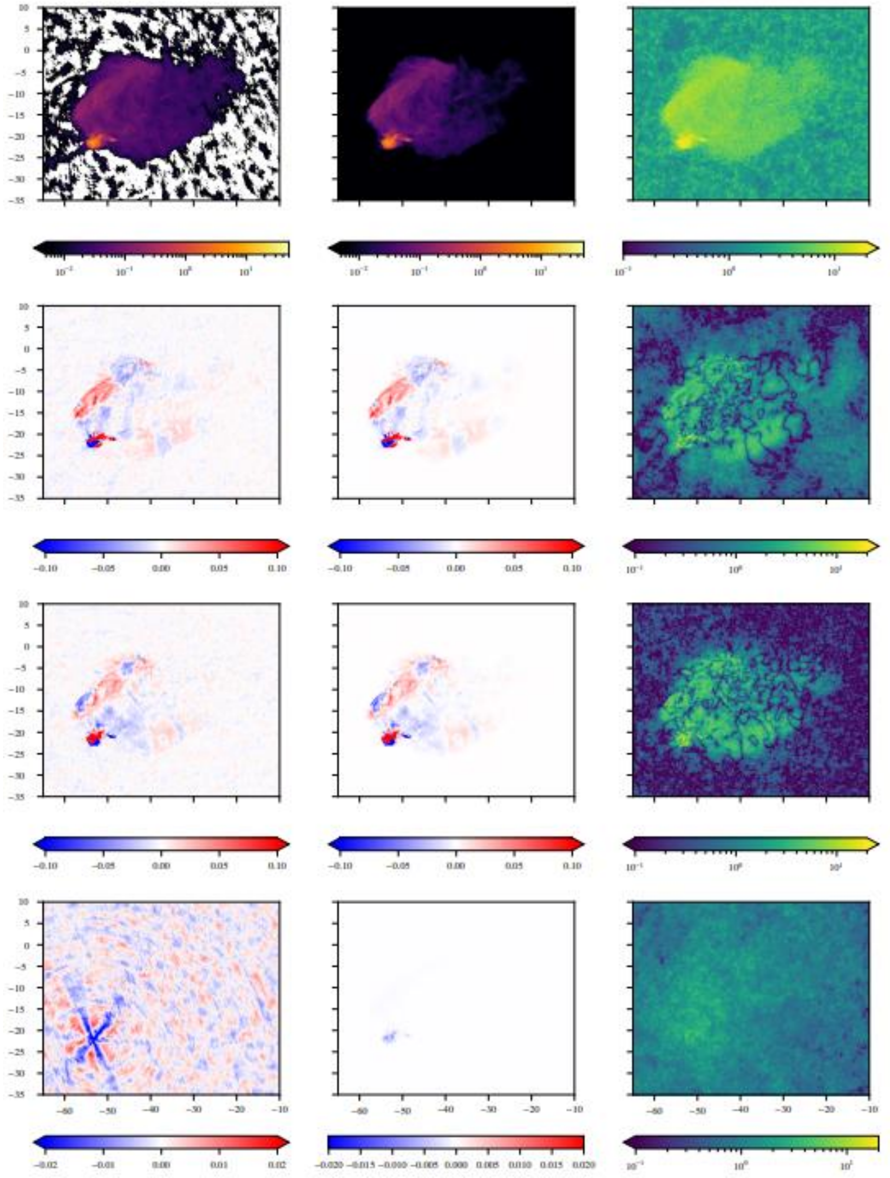
Component detection & separation

Fuchs et al., 2025



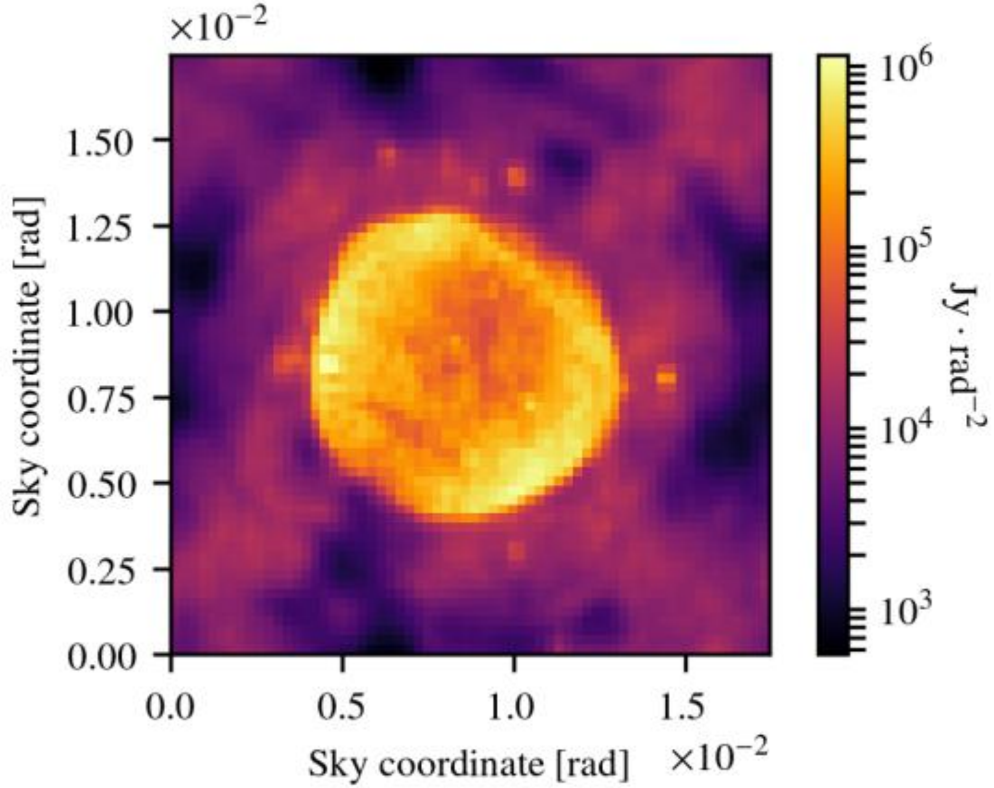
Polarization Imaging

Arras et al., 2025



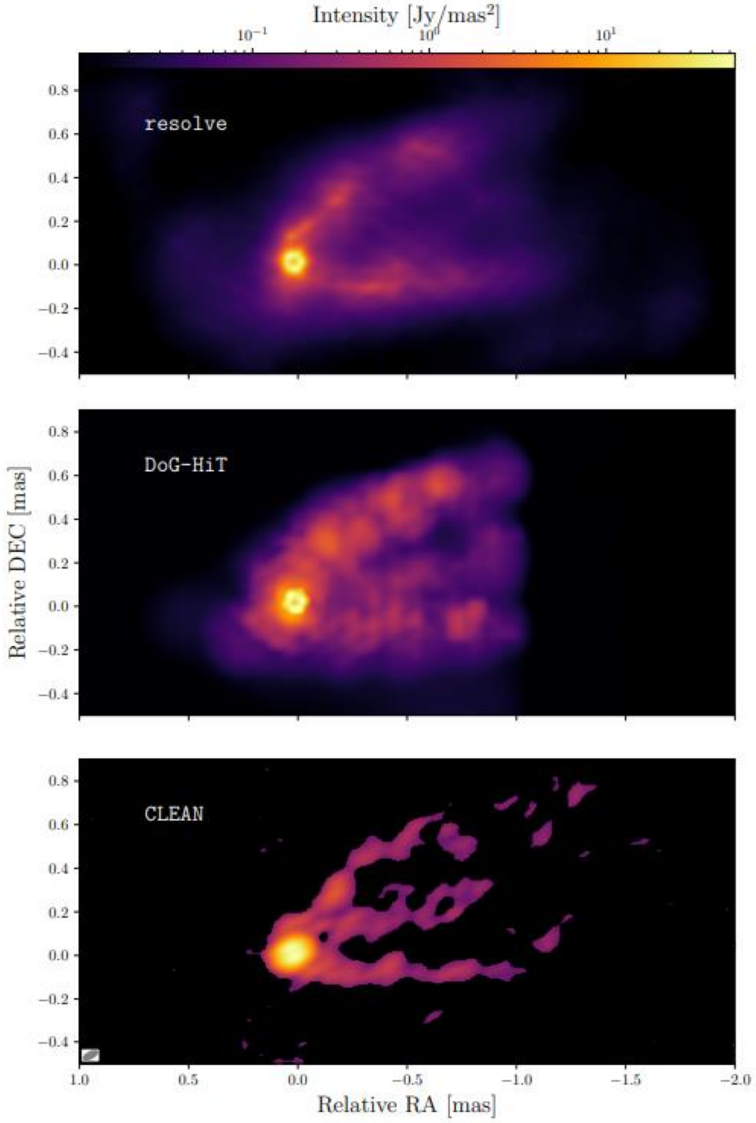
Imaging & Direction Independent Calibration

Arras et al., 2019



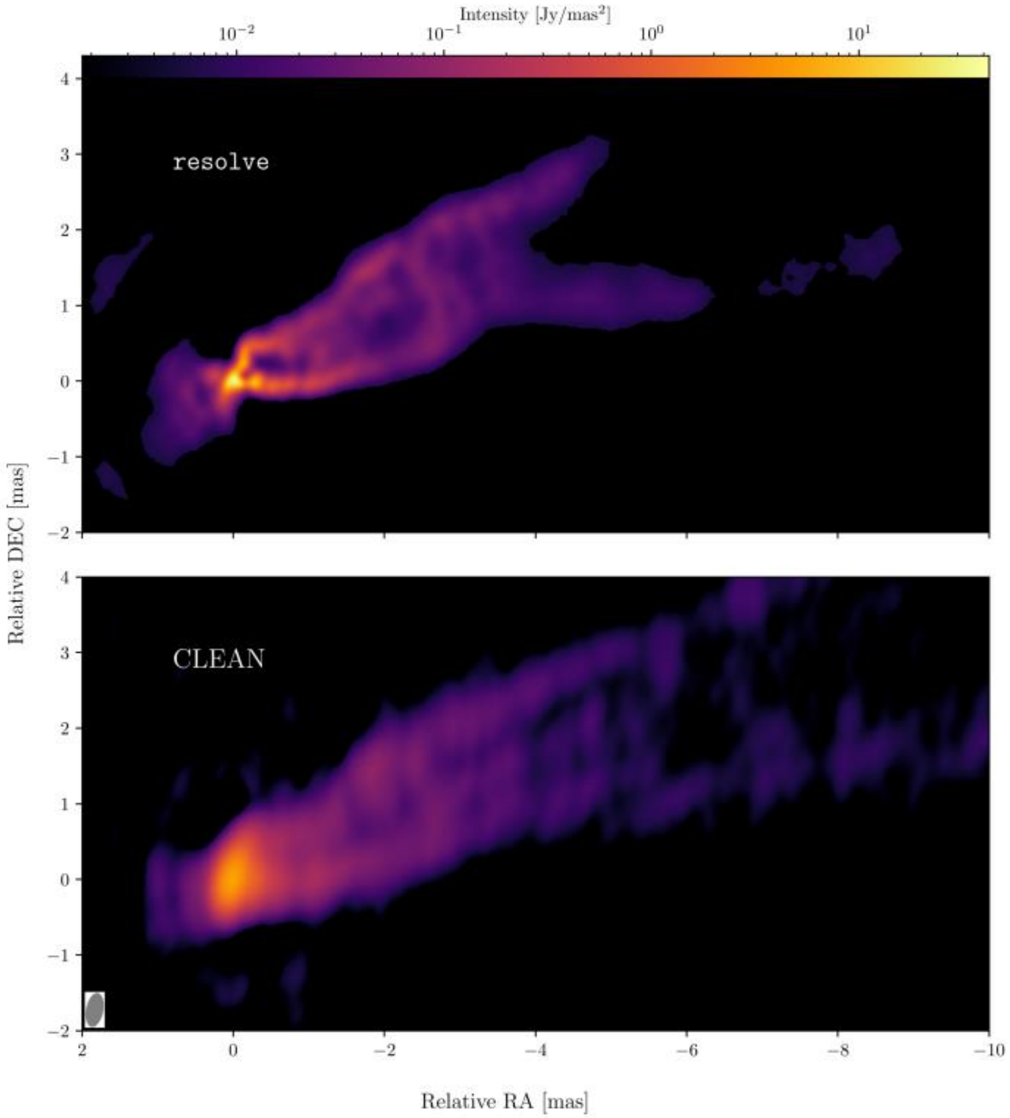
Imaging with VLBI

Kim et al., 2025



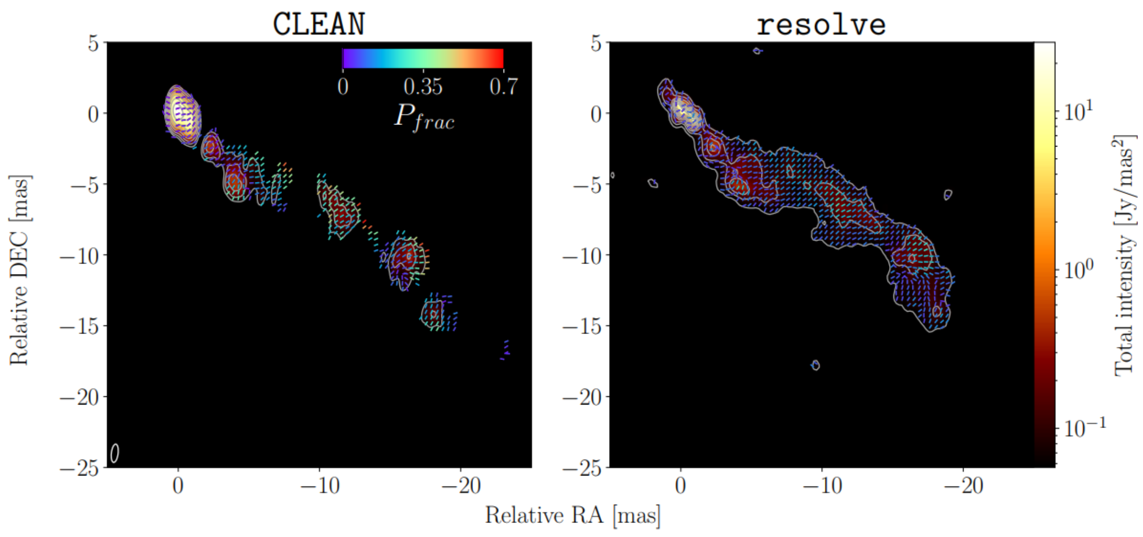
Imaging & Calibration with VLBI

Kim et al., 2024



Polarization Imaging & Calibration with VLBI

Kim et al., 2025



Where to find RESOLVE in the near future?

https://github.com/NIFTy-PPL/J-UBIK